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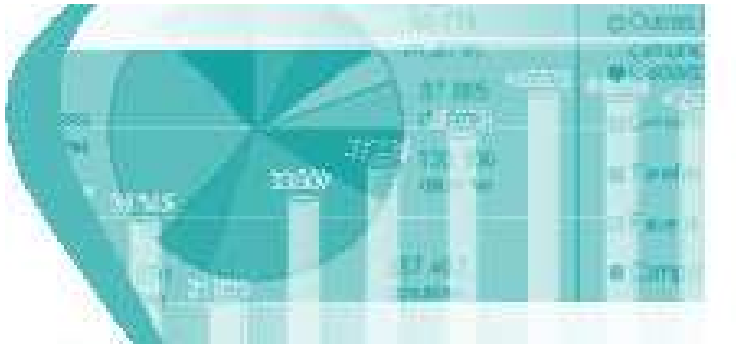
Methodological differences among maturity indicators



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Abstract

Cross country comparisons among public debt maturity indicators must be made with caution. Although several countries report their maturity indicators under the same name, their methodology of calculation may be distinct. Accordingly, it is not possible to establish a direct comparison between indicators of different countries, even if these indicators have the exact same name.

The objective of this report is to analyze the methodological differences among the following indicators: Average Term to Maturity (ATM), Duration, Average Maturity and Average Life.

The Brazilian National Treasury releases regularly two of them: ATM and Average Maturity, being this last one used as reference to year-end targets. On the other hand, the ATM appears to be the indicator most commonly used by other countries.

The comparison among these indicators leads to the conclusion that, despite showing a greater conceptual coherence, the use of Average Maturity as the main debt indicator makes the Brazilian public debt be unfairly penalized when compared with international statistics, which are usually based on ATM.

Methodological differences among maturity indicators

The present report aims at showing that an international comparison among public debt maturity indicators demands caution. Although several countries report use their maturity indicators under the same name, their methodology of calculation may be distinct. Accordingly, it is not possible to establish a direct comparison between indicators of different countries, even if they have the exact same name. Knowing the methodology behind each indicator is essential in order to accomplish a fair comparison.

International literature shows that two indicators are frequently informed in countries' reports: Average Term to Maturity (ATM) and Duration. The Brazilian Treasury[1] debt reports regularly release other two indicators, ATM and Average Maturity, giving more importance to the second one as a guide for monitoring its refinancing risk.

It will become evident, later in this report, that there are methodological differences among these four indicators. These differences reside in the following issues:

- Are the coupon payments taken into account or only principal ones?
- Are the payment flows taken at their face value or present value?
- When the present value is considered, are the flows discounted at market yields or at the issuance yields of each bond or contract?

The following table outlines such differences[2]. In terms of cash flows, ATM and Average Life compute only principal payments, while Duration and Average Maturity take into account both coupon and principal payments. Regarding cash flow valuation, ATM uses face value, while the other three indicators use present value. Finally, when computing the present values, Duration discounts the cash flows at market yields, whilst Average Life and Average Maturity discount them at the issuance yield of each bond or contract.

	Cash flow considered	Valuation	Discount Rate
ATM	Principal	Face Value	-
Duration	Principal + Coupon	Present Value	Market yields
Average Life	Principal	Present Value	Issuance yields
Average Maturity	Principal + Coupon	Present Value	Issuance yields

The use of the ATM is internationally disseminated, although this indicator seems to be inconsistent, since its cash flows are computed based on their face values. In other

[1] For further details, see the Brazilian Annual Borrowing Plan (https://www.tesouro.fazenda.gov.br/en/federal-public-debt/annual-borrowing-plan), Annual Debt Report (https://www.tesouro.fazenda.gov.br/en/publications/annual-debt-report) and Monthly Debt Report (https://www.tesouro.fazenda.gov.br/en/federal-public-debt/monthly-debt-report).

[2] The formulas, followed by a brief methodological description of these indicators, are presented in the appendices.

words, the cash flows are not considered in the same time period. Additionally, there is a distinct treatment, depending on the bond's characteristics. For example, for the fixed rate bonds, their face value at maturity is known beforehand. In the case of indexed bonds, such as inflation-linked or floating rates bonds, the face value is computed at the day of the calculation of the ATM and usually no economic scenario is applied in order to estimate the adjusted face value at maturity. Since different treatments are applied to different types of debt, their cash flows, even if taken at the same point in time, are not comparable.

An indicator that computes the cash flows using the present value, instead of the face value, does not present such fragility. All flows, independently of their associated indexes, are computed by their present values[3].

A numerical example is appropriate to demonstrate how different methodologies lead to distinct magnitudes of maturity. Take a theoretical portfolio composed by three NTN-Fs characterized in the following table. NTN-F is a fixed rate note with coupon payments of 10% p.a. (paid twice a year, in January and July), and principal value of R$ 1,000.00.

	Maturity date	Issuance Yields	Market Yields - april/12
NTNF 010114	01/01/2014	12.782%	9.276%
NTNF 010117	01/01/2017	13.294%	10.319%
NTNF 010121	01/01/2021	13.289%	10.752%

The next table displays the different maturity indicators calculated for that hypothetical portfolio: ATM of 5.2 years, Duration of 4.2 years, Average Maturity of 3.7 years and Average Life of 4.2 years. Therefore, in this example, ATM results approximately 19% higher than the Average Life.

ATM	Duration	Average Maturity	Average Life
5.167	4.195	3.667	4.192

An analyst who does not take into consideration methodological differences will probably make a direct comparison between country A's ATM and country B's Average Life, and will incorrectly infer that country A presents a public debt maturity higher than that of country B, when, in fact, country B's public debt maturity may be equivalent or even superior to country A's. The real picture is polluted by the differences in the adopted methodologies.

[3] An alternative to avoid that variations in forecasts (for inflation, exchange rate and interest rates, etc) affect the evolution of the average maturity is to apply the forecasts when calculating the cash flows' future values, and then compute the cash flows' present value discounting by the same forecasts.

This raises an important concern that deserved a mention in a report issued by the rating agency Moody's[4]. In the fourth page of this report there is a table comparing public debt maturities of several Latin American countries, whose footnote has the following remark: *"Some countries use different methodologies to calculate average maturity. Most countries use a simple weighted average of current market debt values. Brazil uses the present value to calculate the average maturity, which makes the numbers to appear worse than they actually are relative to its peers".*

Latin America: Average Maturity (Years)

TOTAL	2005	2006	2007	2008	2009
Argentina	12.3	12.9	12.6	11.7	11.1
Brazil	4.6	4.9	5.3	5.6	5.5
Chile	9.2	8.2	9.3	11.8	12.1
Colombia	5.2	5.8	5.8	6.9	6.6
Costa Rica	11.1	10.1	9.6	9.9	9.8
D.R.	N/A	N/A	6.5	5.9	5.7
El Salvador	14.2	15.1	14.6	14.0	12.8
Guatemala	11.0	11.8	12.4	12.5	12.7
Mexico	NA	NA	6.4	7.2	7.0
Peru	8.4	8.4	11.3	11.2	11.4
Uruguay	7.9	12.1	13.6	13.0	12.7
Venezuela	N/A	8.5	9.0	9.9	9.5

The following table shows that, in a sample of twelve countries, the most commonly disclosed indicators are ATM and Duration: ten countries disclose ATM and eight disclose Duration. Brazil is the only country that calculates Average Life and Average Maturity, and only Belgium calculates Average Life, associated to Duration.

Country	**Indicator 1**	**Indicator 2**
South Africa	ATM	Duration
Australia	ATM	Duration
Chile	ATM	Duration
Finland	ATM	Duration
Mexico	ATM	Duration
Poland	ATM	Duration
Portugal	ATM	Duration
Czech Republic	ATM	-
Spain*	ATM	-
Italy*	ATM	-
Belgium	Average Life	Duration
Brazil	Average Life	Average Maturity

Source: Ministry of Finance of each country.

* Although these countries name the indicator Average Life, the methodology used in the calculation is equivalent to the ATM.

[4] The report is labeled "Latin America: A Look at Debt Structure and Financing Needs", dated 08/04/2010.

It is important to highlight that the use of issuance yields of contracts or bonds when calculating the Average Maturity, instead of market yields (which would lead to Duration), arises from the combination of greater volatility and level of interest rates in Brazil, when compared to developed economies[5]. If Duration was selected as the maturity indicator for the Brazilian public debt, the high volatility in market yields would make it difficult to analyze the debt maturity evolution through time and to establish a precise target for maturity, as released in Brazil's Annual Borrowing Plan. Hence, the choice of Duration over Average Maturity would tend to hamper the communication between debt managers and its stakeholders, besides not generating major gains in terms of accuracy.

[5] Rossi (2010), entitled "Padrões de integração financeira e a volatilidade das taxas de câmbio e juros em países periféricos", is an example of a research showing that the volatility of Brazilian interest rates is high compared to the standards observed in developed economies.

Appendices

1 – Methodological description of the main indicators

1) Average Time to Maturity - ATM

Because the methodology used by most countries to calculate their maturity indicators considers only principal payments, the Brazilian Treasury decided to calculate the ATM of its debt, aiming at increasing the international comparability of its debt indicators.

ATM calculation is the average remaining time to maturity of each security or contract composing the debt. It is appropriate that this average be weighted by the face value of each security.

It is possible, though, that some countries compute a simple average, which renders the indicator less precise. In an OECD[6] report it is possible to learn about the methodology behind the indicator: "No account is taken of interest payments in the calculation of ATM, (...) no present value calculation is undertaken". Or still, "In the calculation of ATM the weights are the nominal repayments of principal at the date they fall due".

The ATM is also known as the Average Residual Life, which adds to the confusion in terminology, because in translations Average Residual Life may be mistaken for Average Life.

The proposed formula is:

$$ATM = \frac{\sum_{t=0}^{n} t \times P_t}{\sum_{t=0}^{n} P_t}$$

Where t is the maturity (usually measured in years) and Pt is the principal value maturing in t.

2) Duration

This indicator, also known as Macaulay Duration, represents the average maturity of the bonds or contracts, weighted by its cash flows. Such flows, which include coupon payments, are computed at present value, using as discount rates the prevailing market yields at the moment of the calculation.

[6] See page 17 of the document entitled "UNITAR e-Learning Course on Audit of Public Debt - Module 3: Public Debt: Assessing Risk and Sustainability". Available at: http://www.publicdebtnet.org/export/sites/PDM/public/Learning/Audit_of_Public_Debt/Module_3.pdf.

According to the mentioned OECD report, Duration is the statistic for interest rate risk most commonly used in public debt management. In the same document, the formula for Durations is expressed as:

$$D = \frac{\sum_{t=0}^{n} t \times CF_t \times (1+i)^{-t}}{\sum_{t=0}^{n} CF_t \times (1+i)^{-t}}$$

Where CFt represent the cash flow (coupon or principal payments) at the moment "t", and "i" is the market yield used to calculate the present value.

3) Average Maturity

The Average Maturity is almost identical to Duration, but each security's market yield is replaced by its issuance yield when computing the present value.

4) Average Life

The Average Life formula is basically the same of the ATM, but principal flows are discounted at present value using the security's issuance yield:

$$Average\ Life = \frac{\sum_{i=1}^{n} \left(\frac{Flow_i}{(1+IRR)^{\frac{n}{p}}} \times n_i \right)}{\sum_{i=1}^{n} \left(\frac{Flow_i}{(1+IRR)^{\frac{n}{p}}} \right)}$$

Where:

$Flow_i$ = principal payments of bonds or contracts;
n_i = number of days between the cash flow date and the calculation date, based on the adopted day counting standard;
IRR = average issuance yield of the Bond/contract;
n = number of days between the reference date for the calculations and the date of the cash flow;
p = numbers of day in a year, according to the adopted day counting standard of 252 or 360, according to the type of Bond/contract;

2 – ATM formulas for Brazilian domestic government securities

1. Notas do Tesouro Nacional, série F – NTN-F

$$ATM = \frac{\text{t}.1000}{1000}$$

$$ATM = \frac{\sum_{i=1}^{n} \text{t}_i.1000}{\text{n}.1000}$$

Face Value: R$ 1.000,00

2. Letras do Tesouro Nacional, LTN

$$ATM = \frac{\text{t}.1000}{1000}$$

$$ATM = \frac{\sum_{i=1}^{n} \text{t}_i.1000}{\text{n}.1000}$$

Face Value: R$ 1.000,00

3. Letras Financeiras do Tesouro, LFT

$$ATM = \frac{\text{t}.\text{Adjusted Nominal Value}}{\text{Adjusted Nominal Value}}$$

$$ATM = \frac{\sum_{i=1}^{n} \text{t}_i.\text{Adjusted Nominal Value}}{\text{n}.\text{Adjusted Nominal Value}}$$

4. Notas do Tesouro Nacional, série B – NTN-B and NTN-B Principal

$$ATM = \frac{\text{t}.\text{Adjusted Nominal Value}}{\text{Adjusted Nominal Value}}$$

$$ATM = \frac{\sum_{i=1}^{n} \text{t}_i.\text{Adjusted Nominal Value}}{\text{n}.\text{Adjusted Nominal Value}}$$

5. Notas do Tesouro Nacional, série C – NTN-C

$$ATM = \frac{\text{t}.\text{Adjusted Nominal Value}}{\text{Adjusted Nominal Value}}$$

$$ATM = \frac{\sum_{i=1}^{n} \text{t}_i.\text{Adjusted Nominal Value}}{\text{n}.\text{Adjusted Nominal Value}}$$